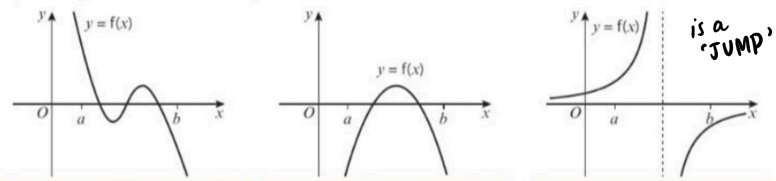
LOCATING ROOTS

· find root of equation + when sure a root lies in stated range

· Root of function is val x when $f(x) = 0$

· If $f(x)$ is continuous on interval $[a,b]$ and $f(a)$ and $f(b)$ are OPPOSITE SIGNS then $f(x)$ has at least one root, x, which satisfies $a < x < b$.

There are three situations you need to watch out for when using the change of sign rule to locate roots. A change of sign does not necessarily mean there is exactly one root. Also, the absence of a sign change does not necessarily mean that a root does not exist in the interval.



There are multiple roots within the interval $[a, b]$. In this case there is an **odd number** of roots.

There are multiple roots within the interval $[a, b]$, but a sign change does not occur. In this case there is an **even number** of roots.

There is a vertical asymptote within interval $[a, b]$. A sign change does occur, but there is no root.

EXAM

1. Find $f(x)$ output for 2 vals range
2. Refer to Δ in sign

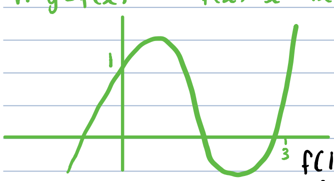
range: $[a,b]$ OR $a<x<b$

Δ sign & $f(x)$=continuous

· Function is continuous if line does NOT 'jump' [∴ can skip past 0 due vertical asymptote]

No sign change: no roots OR even no. roots in interval

1. $y = f(x)$ $\quad f(x) = x^3 - 4x^2 + 3x + 1$



a. explain how graph shows oot bet. $x = 2$ and $x = 3$

graph crosses x-axis bet $x=2$ and $x=3$

∴ root of $f(x)$ lies bet

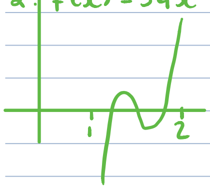
b. Show $f(x)$ has root bet $x = 1{\cdot}4$ and $x = 1{\cdot}5$

$f(1{\cdot}4) = (1{\cdot}4)^3 - 4(1{\cdot}4)^2 + 3(1{\cdot}4) + 1 = 0{\cdot}104, > 0$

$f(1{\cdot}5) = (1{\cdot}5)^3 - 4(1{\cdot}5)^2 + 3(1{\cdot}5) + 1 = -0{\cdot}125, < 0$

↳ There's Δ of sign bet $1{\cdot}4$ and $1{\cdot}5$ ∴ at least one root in interval.

2. $f(x) = 54x^3 - 225x^2 + 309x - 140$



Student observe $f(1{\cdot}1)$ and $f(1{\cdot}6)$ both -ve & states $f(x)$ no roots in interval $[1{\cdot}1, 1{\cdot}6]$

a. Explain why WRONG

diagram shows could be 2 roots interval $[1{\cdot}1, 1{\cdot}6]$

b. Calc $f(1{\cdot}3)$ & $f(1{\cdot}5)$ and ∴ explain why at least 3 roots interval $1{\cdot}1 < x < 1{\cdot}7$:

$f(1{\cdot}1) = -0{\cdot}476, < 0$

$f(1{\cdot}3) = 0{\cdot}088, > 0$

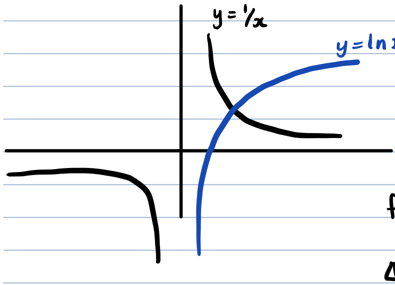
$f(1{\cdot}5) = -0{\cdot}5 \ < 0$

$f(1{\cdot}7) = 0{\cdot}352 \ > 0$

change sign bet $[1{\cdot}1, 1{\cdot}3]$ and $[1{\cdot}3, 1{\cdot}5]$ and $[1{\cdot}5, 1{\cdot}7]$

∴ at least 3 roots interval $1{\cdot}1 < x < 1{\cdot}7$

3. a. Sketch $y = \ln x$ and $y = \frac{1}{x}$ & explain why $y = \ln(x) - \frac{1}{x}$ only 1 root



lines intersect when $\ln x = \frac{1}{x}$ $\left(\ln x - \frac{1}{x} = 0 \right)$

$\therefore$ roots = p.o. intersect and only 1.

b. Show this root lies interval $1.7 < x < 1.8$

$f(x) = \ln x - \frac{1}{x}$ $\quad f(1.7) = -0.0576$, < 0

$f(1.8) = 0.0322$, > 0

Δ of sign bet 1.7 and 1.8 $\therefore$ root $[1.7, 1.8]$

c. Given root $f(x)$ is a, show $a = 1.763$ correct 3dp

bounds : $[1.7625, 1.7635]$

$f(1.7625) = -0.00064 \; < 0$

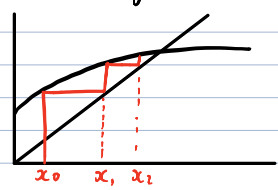
$f(1.7635) = 0.00024 \; > 0$

↳ Δ sign interval $\therefore$ $1.7625 < a < 1.7635$ $\therefore$ $a = 1.763$ 3dp

USE ITERATION TO APPROX. A ROOT

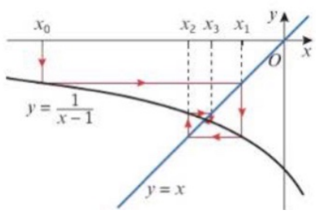
- Solve $f(x) = 0$ by iterative method, rearrange $f(x) = 0$ into $x = g(x)$ & use iterative formula $x_{n+1} = g(x_n)$

- Converge to root : ① Successive iteration get closer to root from same direction



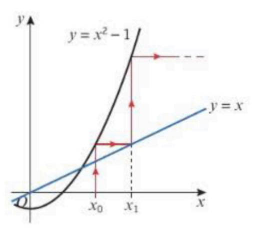
staircase diagram

② Successive it. alternate bet below & above root



cobweb diagram

- Diverges - iteration moves AWAY from root (↑ quick)



$\cos^{-1}$ = arc cos

root ⇝ when $x_0, x_1 \dots$ reach constant

*Trig = always radians